# FEATPOST manual 

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FEATPOST is an extension of the METAPOST language that has a fairly large set of features to facilitate the production of schematic diagrams, both in three dimensions (3D) and in two dimensions (2D).

These schematic diagrams are vectorial and focus on the representation of edges (unlike ray-traced raster images that focus on surfaces).

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## 1 Getting started

input featpost3Dplus2D;

## 2 First taste of featpost

Each perspective depends on the point of view. FEATPOST uses the global variable f, of color type, to store the ( $X, Y, Z$ ) space coordinates of the point of view. Also important is the aim of view (global variable viewcentr). This pair of points defines the line of view.

The perspective consists of a projection from space coordinates into planar $(u, v)$ coordinates on the projection plane. FEATPOST uses a projection plane that is perpendicular to the line of view and contains the viewcentr. Furthermore, one of the projection plane axes is horizontal and the other is perpendicular and on the projection plane. "Horizontal" means parallel to the $X Y$ plane.

One consequence of this setup is that $f$ and viewcentr must not be on the same vertical line. The three kinds of projection known to FEATPOST are schematized in figures 1,2 and 3. The macro that actually does the projection is, in all cases, rp.


Figure 1: Central projection (default).


Figure 2: Parallel projection.


Figure 3: Spherical projection. The spherical projection is the composition of two operations: (i) there is a projection onto a sphere and (ii) the sphere is plaited onto the projection plane.

### 2.1 Bugs

It is important to keep in mind that some capabilities of FEATPOST, although usable, may be considered "buggy" or only partially implemented. These include the drawing of cylinders with holes, as in figure 4.


Figure 4: FEATPOST example containing a rigorousdisc with five holes, four of which are fake.

### 2.2 Moving on, slowly

It is highly beneficial to be able to understand and cope with METAPOST error messages as FEATPOST has no protection against mistaken inputs. One probable cause of errors is the use of variables with the name of procedures (macros), like

$$
X, Y, Z, W, N, r p, c b, p s, ~ v p
$$

All other procedure names have six or more characters.
The user must be aware that METAPOST has a limited arithmetic power and that the author has limited programming skills, which may lead to unperfect 3D figures, very long processing time or shear bugs. It's advisable not to try very complex diagrams at first and it's recommended to keep 3D coordinates near order 1 (default METAPOST units).

All three-dimensional FEATPOST macros are build apon the METAPOST color variable type. It looks like this:

```
(red,green,blue)
```

Its components may, nevertheless, be arbtitrary numbers, like:
(X,Y,Z)

So, the color type is adequate to define not only colors but also 3D points and vectors.
One very minimalistic example program could be:

```
input featpost3Dplus2D;
beginfig(1);
    cartaxes(1,1,1);
endfig;
end.
```

where cartaxes is a FEATPOST macro that produces the Cartesian reference frame with axis labels.

The main variable of any three-dimensional figure is the point of view. FEATPOST uses the variable $f$ as the point of view. Spread is another global variable that controls the size of the projection. Therefore the minimalistic program above should also contain, for example:

```
f:=(6,1,3);
Spread:=40;
```


### 2.3 Why FEATPOST?

FEATPOST is good enough to produce scientific diagrams:

- Figure 1 of Phys. Rev. E, 60, 2985-2989 (1999).
- Figures 4, 6 and 8 of Eur. Phys. J. E, 2, 351-358 (2000)
- Figures 8 and 12 of Eur. Phys. J. E, 20, 55-61 (2006).


### 2.4 A small subset of features

### 2.4.1 Angles

Some problems often require defining angles, and diagrams are needed to visualize their meanings. The angline and squareangline macros support this (see figure 5).


Figure 5: Example that uses cartaxes, squareangline, angline and getangle.

### 2.4.2 Parametric lines

Visualizing parametric lines is another need. When two lines cross, one should be able to see which line is in front of the other. The macro emptyline can help here (see figure 6).


Figure 6: FEATPOST diagram using emptyline.

### 2.4.3 Curved solids

Some curved surface solid objects can be drawn with FEATPOST. Among them are cones (verygoodcone), cylinders (rigorousdisc) and globes (tropicalglobe). These can also cast their shadows on a horizontal plane (see figure 7). The production of shadows involves the global variables LightSource, ShadowOn and HoriZon.


Figure 7: FEATPOST diagram using the macros rigorousdisc, verygoodcone, tropicalglobe and setthestage.

### 2.4.4 Fat sticks

One feature that merges 2D and 3D involves what might be called "fat sticks". A fat stick resembles the Teflon magnets used to mix chemicals. They have volume but can be drawn like a small straight line segment stroked with a big pencircle. Fat sticks may be used to represent direction fields (unitary vector fields without arrows). See figure 8 .

### 2.4.5 From 3D to 2D

The most important macro is $r p$ that converts 3D points to two-dimensional (2D) rigorous, orthogonal or fish-eye projections. To draw a line in 3D-space try


Figure 8: FEATPOST direction field macro director_invisible was used to produce this representation of the molecular structure of a Smectic A liquid crystal.

```
draw rp(a)--rp(b);
```

where a and b are points in space (of color type).
But if you're going for fish-eye it's better to

```
draw pathofstraightline(a,b);
```

If you don't know, leave it as

```
drawsegment(a,b);
```


### 2.4.6 Intersections

The most advanced feature of FEATPOST is the ability to calculate the intersections of planar and convex polygons ${ }^{1}$. It can draw the visible part of arbitrary sets of polygons as in the following program:

```
input featpost3Dplus2D;
numeric phi;
phi = 0.5*(1+sqrt(5));
V1 := ( 1, phi,0);V2 := (-1, phi,0);
V3 := (-1,-phi,0);V4 := ( 1,-phi,0);
V5 := (0, 1, phi);V6 := (0,-1, phi);
V7 := (0,-1,-phi);V8 := (0, 1,-phi);
V9 := ( phi,0, 1);V10:= ( phi,0,-1);
V11:= (-phi,0,-1);V12:= (-phi,0, 1);
makeface1(1, 2, 3,4);makeface2(5,6,7,8);
makeface3(9, 10, 11, 12);
beginfig(1);
```

[^0]

Figure 9: Intersecting polygons drawn with the macro sharpraytrace.

```
    sharpraytrace;
endfig;
end
```

See figure 9 .

### 2.4.7 Coming back to 3D from 2D

It is possible to do an "automatic perspective tuning" with the aid of macro photoreverse. Please, refer both to example photoreverse.mp (see figure 10) and to the following web page: FeatPost Deeper Technicalities,

The idea here is to: (i) have a METAPOST-coded vectorized image; (ii) associate 3D coordinates to a few specific points of the vectorized image; (iii) use photoreverse to obtain the perspective parameters corresponding to the image; and (iv) use those perspective parameters to draw 3D matching schematic diagrams on the image.

### 2.4.8 Coming back to 3D from 1D

Using almost the same algorithm as photoreverse, the macro improvertex allows one to approximate a point in 3D-space with given distances $d$ from three other points (an initial guess $\vec{i}$ is required).

$$
\text { point }:=\operatorname{improvertex}\left(\vec{a}, d_{a}, \vec{b}, d_{b}, \vec{c}, d_{c}, \vec{i}\right)
$$

Approximating a point in 3D-space with given distances from three other points is the same as calculating the intersection of three spheres. And the method to do that is the same as the method to calculate the intersection of a plane, a cylinder and a spheroid (see figure 11.


Figure 10: Example that uses photoreverse. It may not work when vertical lines are not vertical in average on the photo.


Figure 11: Example that uses ultraimprovertex.

### 2.4.9 Scalar function minimization

Macro minimizestep is a minimization routine for scalar functions like $y=f(x)$ where an initial triplet $\left(x_{1}, x_{2}, x_{3}\right)$ with $x_{1}<x_{2}<x_{3}$ is given as a parabolic squeleton that provides a way to search for the smallest value of $y$ (if iterated).

```
point := minimizestep( }\vec{x})(f)
```


## 3 Reference Manual

Some words about notation. The meaning of macro, function, procedure and routine is the same. Global variables are presented like this:

```
vartype var, anothervar
anothervartype yetanothervar
```

Explanation of var, anothervar and yetanothervar. vartype can be any one of METAPOST types but the meaning of color is a three-dimensional point or vector, not an actual color like yellow, black or white. If the meaning is an actual color then the type will be colour. Most of the global variables have default values.

Functions are presented like this:

- returntype function() Explanation of this function. "returntype" can be any one of METAPOST types plus global, draw, drawlabel or MD. "global" means that the function changes some of the global variables. "draw" means that the function changes the currentpicture. "drawlabel" means that the function changes the currentpicture and adds text to it. "MD" means that the returntype is the same as the type of the arguments ( $1,2,3$ or 4 D , that is numeric, pair, color or cmykcolor).

1. type 1 Explanation of the first argument. The type of one argument can be any one of METAPOST types plus suffix or text.
2. type2 Explanation of the second argument. There is the possibility that the function has no arguments. In that case the function is presented like "returntype function".
3. Etc.

### 3.1 Global variables

```
boolean ParallelProj
boolean SphericalDistortion
boolean MalcomX
```

Kind of projection calculated by rp. By default projections are rigorous but if ParallelProj is set true then parallel lines remain parallel in the projection. It is the same as placing the point of view infinitely far without loosing sight. If SphericalDistortion is set true there will be a distortion coming from: (i) the projection being done on a sphere of center $f$ and (ii) this sphere being plaited onto the paper page. When MalcomX is set true, perspectives are calculated with the x coordinate (first coordinate) replaced by the fourth coordinate. The


Figure 12: Figure that uses SphericalDistortion:=true and rigorousdisc.
idea here is to use the fourth coordinate as "time" and visualize yz projections of an animation in a single figure ${ }^{2}$

```
color f, viewcentr
```

The point of view is $f$. The plane or sphere of projection contains the center of view viewcentr. The axis, parallel to zz , that contains the viewcentr is projected on a vertical line.

```
numeric MaxFearLimit
```

The above variable defines the maximum allowed 3D distance between viewcentr and the projection of a point as calculated by rp (remember that 3D distances have no units). Everything located beyond this maximum is compressed into a circumference.

```
numeric Spread
pair ShiftV, OriginProjPagePos
numeric PageWidth
numeric PageHeight
```

These variables control the placement of the projection on the paper. Spread is the magnification and ShiftV is the position of the viewcentr projection on the paper. But, if at some point in your program you introduce produce_auto_scale then the currentpicture will be centered at OriginProjPagePos and scaled to fit inside a rectangle of PageWidth by PageHeight.

```
color V[]
color L[]p[]
color F[]p[]
```

Vertexes, lines and faces. The idea here is to draw polygons and/or arbitrary lines in 3D space. Defining the polygons and the lines can be a bit tedious as FEATPOST is not interactiv $\epsilon^{3}$, First, one defines a list of the vertexes ( $\mathrm{V}[\mathrm{]}$ ) that define the polygons and/or the lines. There is a list of polygons and a list of lines. Each polygon ( $\mathrm{F}[\mathrm{p}[\mathrm{p}]$ ) or line ( $\mathrm{L}[\mathrm{p} \mathrm{p}[\mathrm{]}$ ) is itself a list of vertexes. All vertexes of the same poligon should belong to the same plane.

[^1]```
numeric NL
numeric npl[]
numeric NF
numeric npf[]
```

Number of lines, number of vertexes of each line, number of faces, number of vertexes of each face.

```
numeric PrintStep
```

Printstep is the size of iterative jumps along lines. Used by lineraytrace, faceraytrace and pathofstraightline. Big Printsteps make fast lineraytraceings.

```
boolean FCD[]
colour TableC[]
numeric TableColors
numeric FC[]
colour HigColor
colour SubColor
color LightSource
```

FCD means "face color defined". The draw_invisible macro draws polygons in colour, if it is defined. The colour must be selected from the table of colours TableC that has as many as TableColors. The colour FC of each polygon will depend on its position relatively to LightSource where we suppose there is a lamp that emits light coloured HigColor. Furthermore the colour of each polygon may be modified if it belongs to a functional or parametric surface. In this case, if we are looking at the polygon from below than SubColor is subtracted from its colour.

```
numeric RopeColorSeq[]
numeric RopeColors
```

The above variables are used by ropepattern.

```
numeric TDAtiplen
numeric TDAhalftipbase
numeric TDAhalfthick
```

The above variables control the shape of Three-Dimensional Arrows.

```
boolean ShadowOn
numeric HoriZon
```

When ShadowOn is set true, some objects can cast a black shadow on a horizontal plane of Z coordinate equal to HoriZon (an area from this plane may be drawn with setthestage or with setthearena) as if there is a punctual source of light at LightSource. The macros that can produce shadows, in addition to their specific production, are

- emptyline
- rigorousdisc
- verygoodcone
- tropicalglobe
- positivecharge
- whatisthis
- spheroid
- ellipsoid
- kindofcube
- draw_all_test
- fill_faces
- smoothtorus

All macros that contain shadow in their name calculate the location of shadows using cb. These are: circleshadowpath; signalshadowvertex; ellipticshadowpath; circleshadowpath; spheroidshadow; ellipsoidshadow; torushadow; rigorousfearshadowpath; and faceshadowpath.

```
path VGAborder
```

This path and the macro produce_vga_border are meant to help you clip the currentpicture to a $4: 3$ rectangle as in a (old) movie frame.

```
pair PhotoPair[]
color PhotoPoint[]
numeric PhotoMarks
```

The above variables are used by photoreverse.

```
pen ForePen, BackPen
path CLPath
numeric NCL
```

The above variables are used by closedline.

```
boolean OverRidePolyhedricColor
string ostr[]
numeric ActuC, Nobjects, RefDist[]
```

OverRidePolyhedricColor is used by fillfacewithlight. Nobjects, ostr and RefDist [] are auxiliary variables used by getready and doitnow. Actuc is used both by hexagonaltrimesh and by partrimesh.

### 3.2 Definitions

- global makeline@\#( text1)
- global makeface@\#( text1)

Both of these functions ease the task of defining lines and polygons. Just provide a list of vertexes in a correct sequence for each polygon and/or line. Suppose a tetrahedron

```
V3:=(+1, -1, -1);V2:=(-1,+1, -1);
V4:=(+1,+1,+1);V1:=(-1, -1,+1);
makeface2(1, 2, 3);makeface3(1, 2,4);
makeface1(3,4,1);makeface4(3,4,2);
```

The number in the last makeface or last makeline procedure name must be the number of polygons or lines. All polygons and lines from 1 upto this number must be defined but the sorting may be any of your liking.

### 3.3 Macros

### 3.3.1 Very Basic Macros

- numeric $\mathbf{X}()$ Returns the first coordinate of a point or vector (triplet of color type) if MalcomX is false but returns the fourth coordinate of a tetraplet (of cmykcolor type) if MalcomX is true.
- numeric $\mathbf{Y}()$ Returns the second coordinate of a point or vector. Replaces greenpart.
- numeric $\mathbf{Z}()$ Returns the third coordinate of a point or vector. Replaces bluepart.
- numeric $\mathbf{W}()$ Returns the fourth coordinate of a 4D point or vector. Replaces blackpart.
- cmykcolor makecmyk() Produces a tetraplet from a triplet and a scalar.
- color maketrio() This is, in fact, a projection from 4D into 3D. The single input is a tetraplet and the output is a triplet (the fourth coordinate is discarded). The output triplet takes in consideration the value of MalcomX (see X).
- draw produce_auto_scale The currentpicture is centered in, and adjusted to the size of, an A4 paper page. This avoids the control of Spread and ShiftV.
- string cstr() Converts a color into its string. Usefull in combination with getready.
- string bstr() Converts a boolean expression into its string. Usefull in combination with getready.


### 3.3.2 Vector Calculus

- color $\mathbf{N}()$ Unit vector. Returns black (the null vector) when the argument has null norm. The "N" means "normalized".
- numeric cdotprod() Dot product of two vectors.
- color ccrossprod() Cross product of two vectors.
- numeric ndotprod() Cossine of the angle beetween two vectors.
- color ncrossprod() Normalized cross product of twovectors.
- numeric conorm() Euclidean norm of a vector.
- numeric cmyknorm() Euclidean norm of a 4D vector. Should not be used when MalcomX is true.
- numeric getangle() Angle beetween two vectors.
- numeric getcossine() Cossine of the angle between segment A and segment B , where A connects $f$ and the center of a sphere, and where B contains $f$ and is tangent to that sphere.
- pair getanglepair() Orientation angles of a vector. The first angle (xpart) is measured beetween the vector projection on the XY plane and the X axis. The second angle (ypart) is measured beetween the vector and its projection on the XY plane. This may be usefull to find the arguments of kindofcube
- color eulerrotation() Three-dimensional rotation of a vector. See the figure 20 to visualize the following movement: (i) grab the X component of the vector; (ii) rotate it on the XY plane as much as the first argument; (iii) raise it up as much as the second argument; and (iv) turn it around as much as the third argument.

1. numeric Angle of rotation around the Z component.
2. numeric Angle of rotation around the rotated $Y$ component.
3. numeric Angle of rotation around the two times rotated X component.
4. color Vector to be rotated.

- color randomfear Generates a randomly oriented unit vector.
- MD planarrotation $(\vec{A}, \vec{B}, \theta)=\vec{A} \cos \theta+\vec{B} \sin \theta$
- color rotvecaroundanother Rotates a vector around another.

1. numeric Angle of rotation around the fixed vector.
2. color Vector to be rotated.
3. color Fixed vector.

### 3.3.3 Projection Macros

- pair $\mathbf{r p}()$ Converts spatial positions into planar positions on the paper page. The conversion considers the values of the following global variables: viewcentr, ParallelProj, SphericalDistortion, Spread, ShiftV and MaxFearLimit. When both ParallelProj and SphericalDistortion are false it won't work if either (i) the vectors f-viewcentr and $f-R$ are perpendicular ( $R$ is the argument) or (ii) $f$ and viewcentr share the same $X$ and $Y$ coordinates.

1. color Spatial position.

- pair $\mathbf{v p}()$ Converts spatial directions into planar positions on the paper page. These positions are the vanishing points of those directions. The conversion considers the values of the same global variables as rp.

1. color Spatial direction.

- color $\mathbf{c b}()$ Calculates the position of the shadow of a point. Uses HoriZon and LightSource.

1. color Point position.

- color projectpoint() Calculates the intersection beetween a plane and a straight line. The plane contains a given point and is perpendicular to the line connecting the LightSource and this same point. The line is defined by another given point and the LightSource. Summary: projectpoint returns the projection of the second argument on a plane that contains the first argument. Can be used to draw shadows cast on generic planes.

1. color Origin of the projection plane.
2. color Point to be projected.

- color lineintersectplan() Calculates the intersection beetween a generic plane and a straight line. The plane contains a given point and is perpendicular to a given vector. The line contains a given point and is parallel to a given vector.

1. color Point of the line.
2. color Vector parallel to the line.
3. color Point of the projection plane.
4. color Vector perpendicular to the projection plane.

- numeric ps() Used by signalvertex.


### 3.3.4 Plain Basic Macros

- draw signalvertex() Draws a dot sized inversely proportional to its distance from the viewpoint f.

1. color Location.
2. numeric Factor of proportionality ("size of the dot").
3. colour Colour of the dot.

- path pathofstraightline() When using SphericalDistortion:=true, straight lines look like curves. This macro returns the curved path of a straight line beetween two points. This path will have a greater length ("time") when PrintStep is made smaller.
- draw drawsegment() Alternative pathofstraightline that avoids the calculation of all the intermediate points when SphericalDistortion:=false.
- drawlabel cartaxes() Cartesean axis with prescribed lenghtes and apropriate labels.


Figure 13: Figure that uses signalvertex.

1. numeric Length of the X axis.
2. numeric Length of the $Y$ axis.
3. numeric Length of the Z axis.

- drawlabel orthaxes() Cartesean axis with prescribed lenghtes and prescribed labels.

1. numeric Length of the X axis.
2. label Label of the $X$ axis.
3. numeric Length of the $Y$ axis.
4. label Label of the $Y$ axis.
5. numeric Length of the Z axis.
6. label Label of the Z axis.

- draw emptyline() This procedure produces a sort of a tube that can cross over itself. It facilitates the drawing of, for instance, thick helical curves but it won't look right if the curves are drawn getting apart from the point of view. Please, accept this inconveniance. As like many other FEATPOST macros this one can produce visually correct diagrams only in limited conditions. Can cast a shadow.

1. boolean Choose true to join this line with a previously drawn line.
2. numeric Factor of proportionality ("diameter of the tube"). The tubes are just sequences of dots drawn by signalvertex.
3. colour Colour of the tube border.
4. colour Colour of the tube.
5. numeric Total number of dots on the tube line.
6. numeric Fraction of the tube diameter that is drawn with the tube colour.
7. numeric This is the number of dots that are redrawn with the colour of the tube for each drawn dot with the color of the tube border. Usually 1 or 2 are enough.
8. text This is the name a function that returns a 3D point of the line for each value of a parameter in beetween 0 and 1 .


Figure 14: Figure that uses emptyline. The junction point of two different lines is indicated by an arrow.

- draw closedline() This procedure produces a tube that can cross over itself. It facilitates the drawing of, for instance, thick helical curves but it won't look right as its thickness does not change with the distance from the point of view. The drawing is entirely done in two dimensions, so the tube diameter depends on the global variables ForePen and BackPen. There can be more than one line in a figure but all get the same diameter. When calling closedline() in different figures of the same program you must reinitialize both NCL and Nobjects (because closedline() uses getready()).

1. boolean Value of "the line is closed".
2. numeric Total number of path segments on the tube line.
3. numeric Use 0.5 or more.
4. numeric Use 0.75 or more.
5. text This is the name of a function that returns a 3D point of the line for each value of a parameter in beetween 0 and 1 .

- drawlabel angline() Draws an arch beetween two straight lines with a common point and places a label near the middle of the arch (marks an angle). Note that the arch is not circular.

1. color Point of one line.
2. color Point ot the other line.
3. color Common point.
4. numeric Distance beetween the arch and the common point.
5. picture Label.
6. suffix Position of the label relatively to the middle of the arch. May be one of lft, rt, top, bot, ulft, urt, llft and lrt.

- drawlabel anglinen() The same as the previous function but the sixth argument is numeric: $0=\mathrm{rt} ; 1=\mathrm{urt} ; 2=\mathrm{top} ; 3=\mathrm{ulft} ; 4=\mathrm{lft} ; 5=\mathrm{llft} ; 6=\mathrm{bot} ; 7=\mathrm{lrt}$; any other number places the label on the middle of the arch.
- draw squareangline() This is supposed to mark 90 degree angles but works for any angle value.


Figure 15: Figure that uses anglinen and rigorouscircle.

1. color Point of one line.
2. color Point ot the other line.
3. color Common point.
4. numeric Distance beetween the "arch" and the common point.

- path rigorouscircle() 3D circle. The total "time" of this path is 8 . This small number makes it easy to select parts of the path. The circle is drawn using the "left-hand-rule". If you put your left-hand thumb parallel the circle axis then the other left-hand fingers curl in the same sense as the circle path. This path allways starts, approching the view point, from a point on a diameter of the circle that projects orthogonaly to its axis, and rotating around the axis in the way of the left-hand-rule.

1. color Center of the circle.
2. color Direction orthogonal to the circle (circle axis).
3. numeric Radius of the circle.

- draw tdarrow() Draws a flat arrow that begins at the first argument and ends at the second. The shape of the arrow is controled by the global variables TDAtiplen, TDAhalftipbase, TDAhalfthick. This arrow is drawn on the plane that maximizes the perspective of its width. Also, the tip is contracted when TDAtiplen is larger than the length of the arrow.
- draw tdcircarrow() Draws a flat curving arrow. The curve is a circular arch on a plane. The shape of the arrow is controled both by the global variables TDAtiplen, TDAhalftipbase, TDAhalfthick and by the three last arguments.


Figure 16: Figure that uses tdarrow and tdcircarrow.

1. color Position of the center $(\vec{c})$.
2. color Vector perpendicular to the plane $P$ that contains the arrow (rotation axis $\vec{A}$ ).
3. numeric Curve ray.
4. numeric Arrow starting angle. Note that the angle is measured relative to the axis pointing from $\vec{c}$ to f and projected onto $P(\vec{B})$. The angle is positive when it approaches $\vec{A} \times \vec{B}$.
5. numeric Angular amplitude of the curve (may be negative).

- path twocyclestogether() This macro allows you to draw any solid that has no vertexes and that has two, exactly two, planar cyclic edges. In fact, it doesn't need to be a solid. Just provide the pathes of both cyclic edges as arguments but note that the returned path is polygonal. In order to complete the drawing of this solid you have to choose one of the edges to be drawn immediatly afterwards. This is done automatically by the whatisthis macro for the case of two parallel and concentric ellipses.
- path ellipticpath() Produces an elliptic path in 3D space.

1. color Position of the center.
2. color Major or minor axis.
3. color The other axis.

- drawlabel labelinspace() Draw some 2D picture on some 3D plane (only when ParallelProj:=true). Just Transforms the label in the same way as its bounding box, that is, the same way as two perpendicular sides of its bounding box. This is only exact for parallel perspectives.

1. color Position for the lower-left corner.
2. color Orientation of the picture's bottom edge.
3. color Orientation of the picture's letf edge.
4. text 2D picture's name.

### 3.3.5 Standard Objects

- path goodcirclepath() Another 3D circle macro. More rigorous than rigorouscircle but when the direction ortogonal to the circle is almost orthogonal to the line viewpoint--center it doesn't work correctly. The total "time" of this path is 36 .


Figure 17: Example that uses labelinspace.

1. color Center of the circle.
2. color Direction ortogonal to the circle.
3. numeric Radius of the circle.

- draw spatialhalfsfear() An hemisphere. Doesn't work with $f$ inside it.

1. color Center.
2. color Vector ortogonal to the frontier circle and pointing out of the concavity.
3. numeric Radius of the (hemi)sphere.

- path spatialhalfcircle() And yet another 3D circle macro. Only the visible or the hidden part. This is usefull to mark sections of cylinders or spherical major circles.

1. color Center of the circle.
2. color Direction ortogonal to the circle.
3. numeric Radius of the circle.
4. boolean The visible part is selected with true and the hidden with false.

- draw rigorousdisc() 3D opaque cylinder with/without a hole. Can cast a shadow (without the hole).

1. numeric Ray of an axial hole.
2. boolean Option for completly opaque cylinder (true) or partial pipe (false) when there is no hole. When the cylinder has an hole this option should be true.
3. color Center of one circular base.
4. numeric Radius of both circular bases.
5. color Vector that defines the length and orientation of the cylinder. The addition the third and fifth arguments should give the position of the center of the other circular base.

- draw verygoodcone() 3D cone. Can cast a shadow.


Figure 18: Figure that uses tropicalglobe.

1. bolean Option to draw dashed evenly the invisible edge (true) or not (false).
2. color Center of the circular base.
3. color Direction ortogonal to the circular base.
4. numeric Radius of the circular base.
5. color Position of the vertex

- path rigorousfearpath() 3D sphere. Simple but hard.

1. color Center position.
2. numeric Radius.

- draw tropicalglobe() Globe with minor circles. Can cast a shadow.

1. numeric Number of marked latitudes.
2. color Center position.
3. numeric Radius
4. color Axis orientation.

- draw spheroid() Revolution ellipsoid. Can cast a shadow.

1. color Center position.
2. color Position of one pole relative to the center.
3. numeric Radius

- draw whatisthis() An elliptic frustum. Both edges are elliptic an have the same orientation but one may be greater than the other. Can cast a shadow.

1. color Reference edge center.


Figure 19: Figure that uses spheroid.


Figure 20: Figure that uses and explains kindof cube. Note that the three indicated angles may be used as arguments of eulerrotation.
2. color Major or minor axis.
3. color The other axis.
4. numeric Length of the original cylinder.
5. numeric Edges axis length ratio.

- draw kindofcube() Polyhedron with six orthogonal faces (cuboid).

1. boolean Also draw the invisible edges dashed evenly (true) or do not.
2. boolean The reference point may be a vertex (true) or the center(false).
3. color Reference point.
4. numeric Alpha1.
5. numeric Alpha2.
6. numeric Alpha3.
7. numeric L1. Length of the first side.
8. numeric L2. Length of the second side.
9. numeric L3. Length of the third side.

These arguments are represented in figure 20 .

- draw setthestage() Produces an horizontal square made of squares. Its Z coordinate is defined by HoriZon.

1. numeric Number of squares in each side.
2. numeric Size of each side.

- draw setthearena() Produces an horizontal circle made of circles. Its Z coordinate is defined by HoriZon. Due to the fact that the center of a circle is not on the center of its central perspective projection, this may look a bit strange.

1. numeric Number of circles on a diameter.
2. numeric Diameter.

- draw smoothtorus() Toxic donut (not to be eaten). Produces an error message when $f$ is close to the table. Can cast a shadow.

1. color Center.
2. color Direction orthogonal to the torus plane.
3. numeric Big ray.
4. numeric Small ray.

### 3.3.6 Composed Objects

- draw positivecharge() Draws a sphere with a plus or minus sign on the surface. The horizontal segment of the sign is drawn on the horizontal plane that contains the sphere center. The middle point of this segment is on a vertical plane containing the viewpoint.

1. boolean Selects the sign (true means positive).
2. color Position of the center.
3. numeric Sphere ray.

- draw simplecar() Draws a cuboid and four discs in a configuration ressembling an automobile. The first three arguments of simplecar are the same as the the last seven arguments of kindofcube but grouped in colors.

1. color Center of the cuboid that constitutes the body of the car..
2. color Angles defining the orientation of the car (see kindof cube).
3. color Dimensions of the car.
4. color Characteristics of the front wheels. redpart-distance from the front. greenpartwidth of the front wheels (length of the cylinders). bluepart-wheel ray.
5. color Same as above for the rear wheels

- draw banana() Draws a cylindrical strip with a mark in the middle angle.

1. color Center of the base circle.
2. numeric Radius.


Figure 21: Figure that uses positivecharge, getready and doitnow.


Figure 22: Figure that uses setthearena and simplecar.


Figure 23: Figure that uses banana.
3. color Euler angles for the orientation of the strip (uses eulerrotation as if the cylindrical strip axis is the rotation of $\hat{z}$ ).
4. numeric Length of the cylindrical strip.
5. numeric Angular amplitude of half of the cylindrical strip.

- draw quartertorus() Draws a part of a torus.

1. color Center of the base torus.
2. color Vector indicating the position, relative to the center of the base torus, of the center of the circle obtained by cutting the base torus through a plane containing its axle.
3. color Vector indicating the orientation of another similar cutting plane (the norm of vector has no meaning).
4. numeric Radius of cross-section circles.

### 3.3.7 Shadow Pathes

Please remember that not all shadows are pathes.

- draw signalshadowvertex() Draws the shadow of a signalvertex dot. Used by emptyline.

1. color Location of the light-blocking dot.
2. numeric Factor of proportionality ("size of the dot").
3. colour Colour of the dot.

- path ellipticshadowpath() Produces the shadow of an elliptic path.

1. color Position of the center.


Figure 24: Figure that uses quartertorus.
2. color Major or minor axis.
3. color The other axis.

- path circleshadowpath() Produces the shadow of a circle.

1. color Center of the circle.
2. color Direction ortogonal to the circle.
3. numeric Radius of the circle.

- path rigorousfearshadowpath() 3D sphere shadow.

1. color Center position.
2. numeric Radius.

### 3.3.8 Differential Equations

Before we proceed, be aware that solving differential equations ( DE ) is mainly an experimental activity. The most probable result of a procedure that atempts to solve a DE is garbage. The procedure may be unstable, the solution may be littered with singularities or something may go wrong. If you don't have a basic understanding of differential equations then skip this section, please.

- path fieldlinepath() A vectorial field line is everywhere tangent to the field vectors. Two different parallel fields have the same field lines. So the field only constrains the direction of the field lines, not any kind of "speed" and, therefore, it is recommended to normalize the field before using this macro that contains a second-order Runge-Kutta method implementation.

1. numeric Total number of steps.
2. color Initial position.
3. numeric Step (arc)length.
4. text Name of the function that returns a field vector for each 3 D position.

- path trajectorypath() The acceleration of a particle in a conservative force field is equal to the ratio (conservative force)/(particle mass). The acceleration is also equal to the second order time derivative of the particle position. This produces a second order differential equation that we solve using a second-order Runge-Kutta method implementation.

1. numeric Total number of steps.
2. color Initial position.
3. color Initial velocity.
4. numeric Time step.
5. text Name of the function that returns a (force/mass) vector for each 3 D position.

- path magnetictrajectorypath() The acceleration of a charged particle in a magnetic field is equal to the ratio (magnetic force)/(particle mass) but the magnetic force depends on both the velocity and the magnetic field. The acceleration is also equal to the second order time derivative of the particle position. This produces a second order differential equation that we solve using a fourth-order Runge-Kutta method implementation.

1. numeric Total number of steps.
2. color Initial position.
3. color Initial velocity.
4. numeric Time step.
5. text Name of the function that returns a (charge)*(magnetic field)/(partcle mass) vector for each 3 D position.

### 3.3.9 Renderers

- draw sharpraytrace Heavy procedure that draws only the visible part of all edges of all defined faces. There's no point in using this procedure when there are no intersections beetween faces. Any how this will not work for non-convex faces nor when SphericalDistortion:=true.
- draw lineraytrace() Draws only the visible part of all defined lines using sequences of dots (signalvertex and PrintStep).

1. numeric Dot size.
2. colour Dot colour.

- draw faceraytrace() Draws only the visible part of all edges of all defined faces using sequences of dots (signalvertex and PrintStep).

1. numeric Dot size.

## 2. colour Dot colour.

- draw draw_all_test() Draws all defined edges (and lines) in a correct way independently of the kind of projection used. Can cast a shadow (but the shadow is not correct when SphericalDistortion:=true).

1. boolean If true the lines are also drawn.

- draw fill_faces() Unfills and draws all faces in the order they were defined (without sorting). Can cast a shadow.

1. text Like the argument of drawoptions but used only inside this macro and only for the edges.

- draw draw_invisible() This is a fast way of removing hidden lines that doesn't allow for intersecting polygons nor polygons of very different area. It works by + sorting all polygons by distance to $f$ and then by "filling" the polygons. This routine may be used to draw graphs of 3D surfaces.

1. boolean If true polygons are sorted relatively to nearest vertex and, if false, relatively to their mass center. Choose false for surface plots.
2. boolean If false then the polygons are painted with their FC colour modified by LightSource. If true then the next two arguments are used and the polygons are darkened proportionaly to their distance from $f$.
3. colour Colour of faces.
4. colour Colour of the edges.

- global getready () When you don't want to edit the source of the METAPOST program, to resort the objects so they'll be drawn correctly, use this macro and the next.

1. string Command line that would draw some object.

For instance: "draw rigorousfearpath(black,1);".
2. color Reference position of that object.

- draw doitnow The reference positions given as arguments of previous getready calls are used to sort and draw the objects also given as string arguments to previous getready calls. Remember to initialize Nobjects:=0; before a second figure.


### 3.3.10 Nematics (Direction Fields)

Nematics are the least ordered liquid crystals. Their configurations can be described by direction fields (vector fields without arrows). The two following routines ease the task of representing their configurations.

- global generatedirline() Defines a single straight line segment in a given position and with a given orientation.

1. numeric Line index number.
2. numeric Angle beetween the X axis and the projection of the line on the XY plane.


Figure 25: Figure that uses director_invisible and generatedirline.
3. numeric Angle beetween the line and the XY plane.
4. numeric Line (arc)length.
5. color Position of the line middle point.

- draw director_invisible() This is a direction field renderer that can sort direction lines. This routine draws straight lines of given "thickness" beetween the first all the points of all the L[]p[] lines. It is supposed to help you draw vector fields without arrows but taking care of invisibility. The lines may be generated by generatedirline or by other macros.

1. boolean When there is no need to sort lines you may use false here.
2. numeric "Thickness" of the direction lines
3. boolean Use true for cyclic "direction" lines.

### 3.3.11 Surface Plots

FEATPOST surface plots are geared towards unusual features like equilateral triangular grid, hexagonal domain and merging together functional and parametric surface descriptions.

- draw hexagonaltrimesh() Plots a functional surface on a triangular or hexagonal domain. Uses the LightSource.

1. boolean Select the kind of domain. true for hexagonal and false for triangular. The domain is centered on the origin (black). When the domain is hexagonal two of its corners are on the -YY axis. When the domain is triangular one of its corners is on the X axis.
2. numeric Number of small triangles on each side of the triangular domain or three times the number of small triangles on each side of the hexagonal domain.
3. numeric Length of the triangular domain side or three times the hexagonal domain side.
4. text Name of the function that returns the Z coordinate of a surface point of coordinates X and Y .


Figure 26: Figure that uses hexagonaltrimesh.

- global partrimesh() Defines a parametric surface that can be drawn with draw_invisible. In the following descriptions S and T are the parameters. Remember to initialize NF. The surface is defined so that quadrangles are used whenever possible. If impossible, two triangles are used but their orientation is selected to maximize the surface smoothness. Also note that, unlike hexagonaltrimesh(), the spatial range you require to be visible is always first reshaped into a cube and second compressed or extended vertically. How much the cube is compressed or extended depends on the last numeric argument, the compression factor for Z , meaning that the final height of the cube is $2 /$ (compression factor). Thanks to Sebastian Sturm for pointing the need to explain this.

1. numeric Number of $T$ steps.
2. numeric Number of $S$ steps.
3. numeric Minimal $T$ value.
4. numeric Maximal $T$ value.
5. numeric Minimal S value.
6. numeric Maximal S value.
7. numeric Minimal $X$ value.
8. numeric Maximal $X$ value.
9. numeric Minimal $Y$ value.
10. numeric Maximal $Y$ value.
11. numeric Minimal $Z$ value.
12. numeric Maximal Z value.
13. numeric Compression factor for Z values.
14. text Name of the function that returns a surface point (of color type) for each pair ( $\mathrm{S}, \mathrm{T}$ ).

### 3.3.12 Strictly 2D

- path springpath()

1. pair Start point.
2. pair Finish point.
3. numeric Number of swings.
4. numeric Half-width of the swings.
5. numeric Fraction of the length that is occcupied with swings.

- path+draw zigzagfrontier()

1. pair Start point.
2. pair Finish point.
3. numeric Number of swings.
4. numeric Standart deviation of the swings' amplitude.
5. numeric Average swings' amplitude.
6. numeric Outside thickness.
7. numeric Inside thickness.
8. colour Outside color.
9. colour Inside color.

- path randomcirc()

1. numeric Average radius.
2. numeric Standart deviation.
3. numeric Number of points.

- pair radialcross() Calculates one of both intersections between to circles.

1. pair Center of the first circle.
2. numeric Radius of the first circle.
3. pair Center of the second circle.
4. numeric Radius of the second circle.
5. boolean Choice between the upside (true) or downside (false) intersection (relative to the segment connecting both centers).

- draw ropepattern() Draws a (climbing) rope over a path (see figure 27).

1. path The path.
2. numeric Width or thickness of the rope.
3. numeric Number of windings of each thread.

- pair firsttangencypoint() Returns the first point on a path for which the segment connecting that point and another given reference point is tangent to the path.

1. path The path.
2. pair The reference point


Figure 27: Figure that uses ropepattern.
3. numeric Reciprocal of the sampling step along the path in default units.

- path lasermachine() Shrink or swell a cyclic path without cusp points and without coinciding pre and post control points.

1. path The original path.
2. numeric Directed distance between the original path and the returned path (positive values are to the right and negative values are to the left of the original path).
3. numeric Maximum cossine of corner angle above which it remains a sharp corner (only for negative values of directed distance).

- path crossingline() Produces a single path out of two intersecting and cyclic paths. The paths must be adapted with startahead and/or reverse so that they both rotate in the same direction and they start on consecutive "lobes". Now pay attention: given the direction of rotation (clockwise or counter-clockwise) the SecondPath must start BEFORE the FirstPath. And another problem: there must be at least four intersection points.

1. path FirstPath.
2. path SecondPath.
3. numeric Time resolution.

## 4 Missing documentation

```
minimizestep( expr Abcisscolor )( text PlainFunc )
improvertex( expr VerA, DisA, VerB, DisB, VerC, DisC, IniV )
ultraimprovertex( expr PlanPoi, PlanDir, BaseCenter, Radius, LenVec,
```

CentrPoi, NorthPoleVec, Ray, IniV )
necplusimprovertex ( expr PlanPoi, PlanDir, CentrPoiA, NorthPoleVecA, RayA,

CentrPoiB, NorthPoleVecB, RayB, IniV )
intersectprolatespheroid( expr CentrPoi, NorthPoleVec, Ray, LinePoi, LineDir, IniV )
intersectorus( expr Tcenter, Tmoment, Bray, Sray, LinePoi, LineDir )
ellipsoid( expr Centr, AxOne, AxTwo, AxThr )
revolparab( expr BaseCenter, ParabTip, BaseRay )
pointinsidetorus( expr Point, Tcenter, Tmoment, Bray, Sray )

## 5 Reference-at-a-glance

### 5.1 Sphere

tropicalglobe $(N, \vec{c}, R, \vec{A})$


### 5.2 Disc

rigorousdisc ( $R_{i}$, bool, $\vec{c}, R_{o}, \vec{A}$ )

rigorousdisc( 0.5, true, black, 1, 0.85blue );

### 5.3 Torus

smoothtorus $\left(\vec{c}, \vec{A}, R_{b}, R_{s}\right)$


### 5.4 Bowl

spatialhalfsfear $(\vec{c}, \vec{A}, R)$


### 5.5 Cuboid

kindofcube(bool, bool, $\vec{o}, \alpha_{1}, \alpha_{2}, \alpha_{3}, l_{1}, l_{2}, l_{3}$ )

kindofcube( false, true, black, 130, 32, 67, 0.3, 0.6, 0.9 );

### 5.6 Simple car

$\operatorname{simplecar}\left(\vec{o},\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right),\left(l_{1}, l_{2}, l_{3}\right),(\mathbf{X f}, \mathbf{Y f}, \mathbf{Z f}),(\mathbf{X r}, \mathbf{Y r}, \mathbf{Z r})\right)$

simplecar( black, black, (0.8,0.35,0.18), (0.1,0.2,0.132), (0.06,0.06,0.1) );

### 5.7 Cone

verygoodcone( bool, $\vec{c}, \vec{A}, R, \vec{v}$ )

verygoodcone( true, black, blue, 0.8, blue+green );

### 5.8 Elliptic prism

whatisthis $\left(\vec{c}, \vec{S}_{1}, \vec{B}_{1}, D,\left\|\vec{S}_{2}\right\| /\left\|\vec{S}_{1}\right\|\right)$

whatisthis( black, 0.5red, green, 0.85, 0.8 );

### 5.9 Spheroid

$\operatorname{spheroid}(\vec{c}, \vec{S}, R)$

spheroid( black, 2*blue, 1);

### 5.10 Cylindrical strip

$\operatorname{banana}\left(\vec{c}, R,\left(\alpha_{M}, \beta_{M}, \gamma_{M}\right), L, \theta\right)$

5.11 Torus' slice quartertorus $(\vec{c}, \vec{A}, \vec{B}, R)$

quartertorus( black, -red, red-green, 0.25 );

## 6 References

1. "The METAFONTbook" by Don Knuth
2. "METAPOST, a users manual" by John Hobby and the MetaPost development team
3. "The NURBSbook" by Les Piegl and Wayne Tiller

## 7 Acknowledgements

Many people have contributed to make FEATPOST what it is today. Perhaps it would have never come into being without the early intervention of Jorge Bárrios, providing access to his father's computer in 1986. Another important moment happened when José Esteves first spoke about METAPOST sometime in the late nineties.

Also, the very accurate criticism of Cristian Barbarosie has significantly contributed to fundamental improvements. Jens Schwaiger contributed new macros. Pedro Sebastião, João Dinis and Gonçalo Morais proposed challenging new features.


[^0]:    ${ }^{1}$ Unfortunately, this is also the most "bugged" feature.

[^1]:    ${ }^{2}$ To be developed in future versions.
    ${ }^{3}$ The lines could become the skeleton of NURBS.

